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Accessible and Intuitive Mathematical Notation

Anthony Christiana '22

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Towards Notation to Serve Students

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Introduction

Mathematics relies heavily on the use of written symbolic notation. And though the specifics of notation are occasionally debated among mathematicians, most agree that the practice of written mathematics is indispensable for its ability to convey a large amount of information in relatively little space.

However, for math learners with disabilities, engagement with traditional mathematical notation can be difficult or impossible. Here, we explore two examples of the ways that notation can fail disabled students, including discussions on notational inaccessibility, nonintuitivity, and misuse/abuse. Plus, we note how accommodating for these disabilities allows us to imagine better ways of serving *all* students through notational practices.

Print Related Disabilities

Issues

Visual impairment and other forms of print related disabilities often preclude students from accessing written math:

- Most standard screen readers can't read math notation [2][9]
- Classroom math lecturers often omit verbalization when notation conveys meaning
  - Think *action on notation*: "circling like terms" or "crossing out"
- Tactile teaching aids are often replaced by strict formal notation in higher level math

Considerations for Access

MathML

- Is a web-based markup language (think HTML) that displays math on the web[9]
- Is the easiest way to type up math for compatibility with screen readers [2]

Tactile Graphs

- Can be built from common classroom materials
- 3D printing can allow effective and cheap tactile graphs with high precision

Nemeth Code

Nemeth Code is a notational system for mathematics and science using the standard 6-dot Braille system. Math notation extends up to the reserch level.

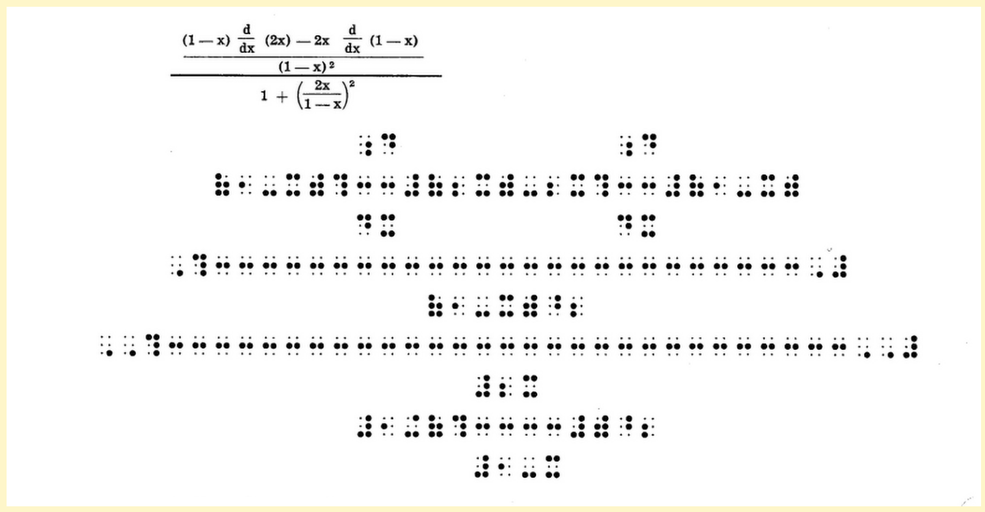

Figure 1: A complex fraction written in Nemeth Code[8]

Learning Disabilities

Issues

Students with learning disabilities like dyslexia, dyscalculia, and ADHD are particularly underserved by mathematics notation:

- Approximately 6% of school-age children have significant math deficits
- Among students classified as learning disabled, arithmetic difficulties are as pervasive as reading problems"
- Students with learning disabilities often bring a solid intuitive understanding of math concepts
- The bridge to formal notation is often where students fall behind [6]

Considerations for Access

Making Convention Explicit

- Interpretation of letters and numbers highly context-driven
- Making these concepts explicit can help students bridge intuition-to-notation gap [1]

Reduce Symbol Density

- Studies find symbol density and lack of symbol familiarity to be major hurdle to understanding [5]
- Visualization and verbalization shown to be more effective

Nonintutive Notation

The Truth is...

Mathematical notation in its current conception and application doesn't just fail in being accessible to students with disabilities.

Notation is:

1. often unintuitive in its design, and
2. taught in a way that makes it seem arbitrary.

These two problems can compound upon each other to create a math learning experience that feels arcane and unsatisfying.

> *In practice, the curriculum is not even so much a sequence of topics, or ideas, as it is a sequence of notations. Apparently mathematics consists of a secret list of mystical symbols and rules for their manipulation. Young children are given '$+$' and '$\div$.' Only later can they be entrusted with '$\sqrt{}$,' and then 'x' and 'y' and the alchemy of parentheses. Finally, they are indoctrinated in the use of '$\sin$,' '$\log$,' '$f(x)$,' and if they are deemed worthy, 'd' and '$\int$.' All without having had a single meaningful mathematical experience.*
>
> - A Mathematician's Lament

Imaging More Intuitive Notation

Mathematical notation is design! So let's ask: *how can we embed the function of symbols into their visual representations?*

To start, we can look to students who have trouble with or question the existing notational system and honor their thinking, speculating on improvements.

$$x^{-1} = (x)^{-1} \not\Rightarrow f^{-1}(x) \neq (f(x))^{-1}$$

$$5(a+b) = 5a + 5b \not\Rightarrow \sin(a+b) \neq \sin(a) + \sin(b)$$

Figure 2: Contextual notation

Figure 3: Glossary of Abuse of Notation

Research has shown that the design of symbols can affect legibility of concepts in math:

	Group 1	Group 2
Addition	◇	▷
Subtraction	▷	◇
Multiplication	◆	▶
Division	▶	◆

Figure 4: Vertically symmetric symbols imply commutativity.

Conclusion: People-First Notation

The Upside?

Considering how to meet the needs of everyone through accessible and intuitive notation can feel daunting, but there is an upside:

Thinking about accessibility will lead to notational practices that best serve *all* math learners.

Some of the accommodations mentioned here are specific to the needs of students with disabilities, but many of them actually reveal the ways in which math notation, and the education thereof, underserves students across the board.When we understand math notation as a set of practices and symbols used to assist math making and not a fixed end goal, we can consider what works and what doesn't. Thinking more critically about making math notation accessible will lead to making math notation better!

References

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[8] Rebecca J. Rosen. The beautiful code, Oct 2013.

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